Mathematics of traffic

“The unreasonable effectiveness of Mathematics”

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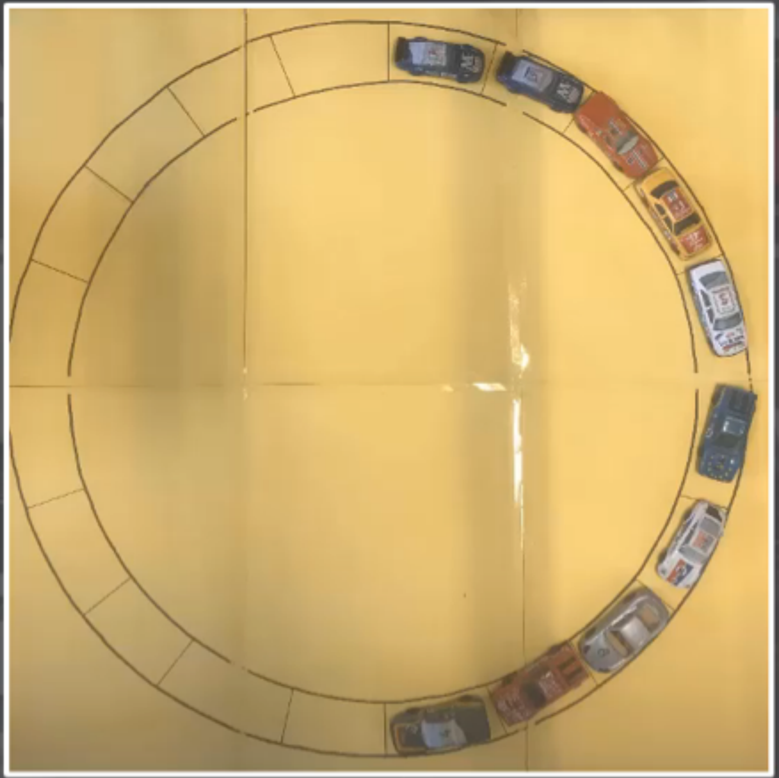
Who has never wondered about how traffic jams start?

Simple modelling of the problem

- Discrete time and space
- *n* cars (*n*=10)
- *2n* sites (*2n*=20)
- max one car per site
- Deterministic parallel dynamics:
  Every iteration each car will move if next site is empty



The current (J) is defined as the number of cars free to move divided by the number of total sites.

$$J(\sigma) = \frac{n^\circ\ of\ cars\ free\ to\ move}{n^\circ\ of\ total\ sites}$$

$$\mathrm{J} = \frac{1}{2}$$

The number of possible configurations is the number of combinations of n elements from a set of 2n.

$$n^\circ\ of\ configurations = \binom{2n}{n}$$

Probabilistic Parallel Dynamics

Serial dynamics variation

The serial dynamics variation is an easier but equivalent way to define the model.
It works as follows:

1. Select **one** site uniformly at random;
2. If there’s a car, move it forward if the next site is empty.

In this case we are able to write the transition matrix that represents the probability to go from every configuration to every other.
This matrix is doubly Markov.

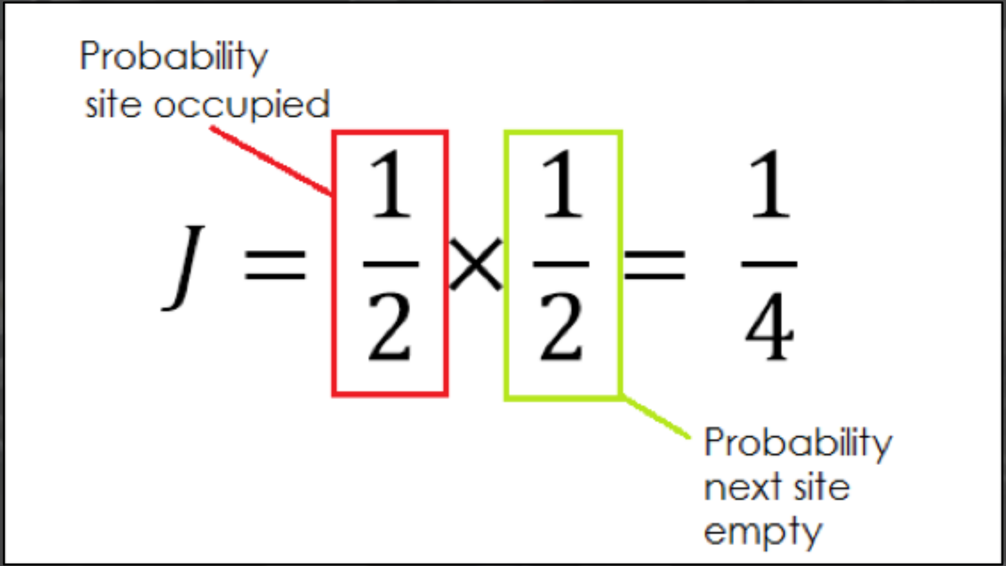
↓

Same probability to observe each state i.e. the **stationary measure is uniform**.

$$\mathbf{P}=\begin{bmatrix} p_{AA} & p_{AB} & p_{AC} & p_{AD} \\ p_{BA} & p_{BB} & p_{BC} & p_{BD} \\ p_{CA} & p_{CB} & p_{CC} & p_{CD} \\ p_{DA} & p_{DB} & p_{DC} & p_{DD} \end{bmatrix}$$

Stationary current

Since every configuration is equally probable, the current is simply the product of the two (independent) probabilities of having a site occupied by a car and the next one empty.



$$J = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Blockage

What happens if we put a localized blockage?

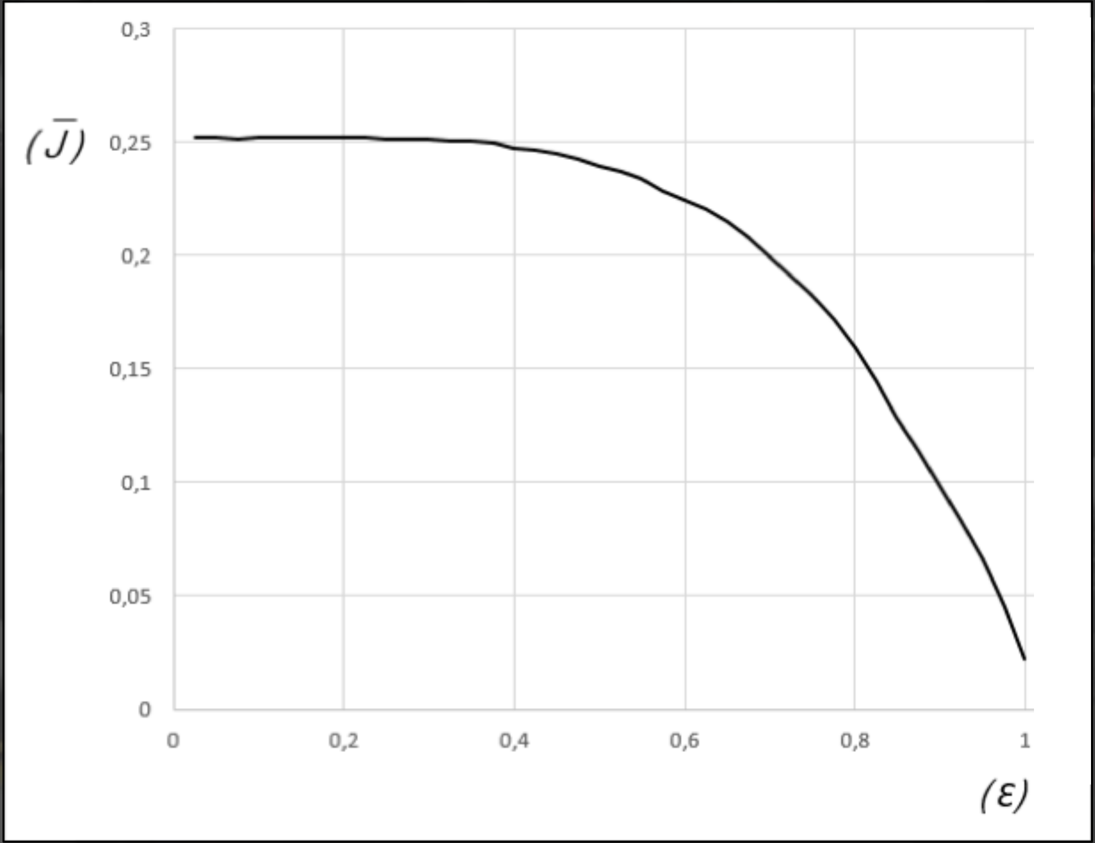
A point where the jump probability is decreased by $\boldsymbol{\varepsilon} > 0$.

Particle-hole symmetry

In the simple case of the deterministic parallel dynamics we observe that a particular symmetry is always conserved:
if a site is empty the site at the opposite side of the ring is occupied and viceversa.

Numerical simulation

```
def j(s)
l=0
s.each_with_index do |k,i|
succ=i+1
succ=0 if i==(s.length-1)
l=l+1 if k==1 and s[succ]==0
end
return l/(1.0*s.length)
end
T=10**2
s = [0,0,1,1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,1,0]
epsilon=0
41.times do
corrente=[]
T.times do |t|
semaforo=1
pos= Random.rand(s.length)
succ= pos+1
if pos==(s.length-1)
  succ=0
  semaforo=0 if Random.rand<epsilon
end
s[pos]= 0 and s[succ]= 1 if s[pos]==1 and s[succ]=
corrente.push(j(s))
end
puts "epsilon=#{(epsilon).round(5)}\tJ=#{(corrente
epsilon+=0.025
end
```



Open questions

1- Does a critical value exist for the blockage intensity ε?

2- What's the nature of the plateau of J(ε) around ε=0?

3- Will some symmetry be conserved with more traffic lights?

Conclusions